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Plane Symmetric String Cosmological Models in Zero-Mass Scalar Fields

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Abstract Non-static plane symmetric cosmological solutions are studied with zero mass scalar fields in the context of cosmic strings in general relativity. Some physical and geometrical features of these models are discussed.

Keywords Strings · Scalar fields · Plane symmetric metric

1 Introduction

The study of interacting fields, one of the fields being a zero-mass scalar field, is basically an attempt to look into the yet unsolved problem of the unification of gravitational and quantum theories. Considerable interest has been focused on a set of field equations representing zero-mass scalar fields coupled with the gravitational field for the last few decades. Bergman and Leipnik [1], Bramhacary [2], Das [3, 4], Stepehenson [5], Gautreau [6], Rao et al. [7], Singh [8], Patel [9], Reddy [10], Venkateswarlu and Reddy [11], Pradhan et al. [12] are some of the authors who have studied various aspects of interacting fields in the framework of general relativity.

In recent years there has been lot of interest in the study of cosmic strings. Cosmic strings have received considerable attention as they are believed to have served in the structure formation in the early stages of the universe. Cosmic strings may have been created during phase transitions in the early era [13] and they act as a source of gravitational field [14]. It is also believed that strings may be one of the sources of density perturbations that are required for the formation of large scale structures of the universe.

The study of cosmic strings in relativistic framework was initiated by Stachel [15] and Letelier [14, 16]. Krori et al. [17, 18], Raj Bali and Shuchi Dave [19], Bhattacharjee and

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Baruah [20], Mahanta and Abhijit Mukharjee [21], Rahaman et al. [22], Reddy [23], Pant and Oli [24] and Venkateswarlu et al. [25] are some of the authors who have studied various aspects of string cosmologies in general relativistic theory as well as in alternative theories of gravitation.

In this paper, we consider non-static plane symmetric space-time in the presence of zero-mass scalar field associated with massive strings as a source.

2 Field Equations and the Metric

The Einstein field equations corresponding to interacting zero-mass scalar fields are given by

$$R_{ij} - \frac{1}{2}g_{ij}R = -T_{ij} - \left(\phi_{,i}\phi_{,j} - \frac{1}{2}g_{ij}\phi_{,k}\phi^{,k}\right) \tag{1}$$

and

$$\phi^{k}_{;k} = 0, \tag{2}$$

where T_{ij} is the total energy momentum tensor for a cloud of massive strings that can be written as

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j. \tag{3}$$

Here ρ is the rest energy density of the cloud of strings with particles attached to them, λ is the tension density of the strings and $\rho = \rho_p + \lambda$, ρ_p being the energy density of the particles. The velocity u^i describes the 4-velocity which has components (1, 0, 0, 0) for a cloud of particles and x^i represents the direction of string which will satisfy

$$u^i u_i = -x^i x_i = 1 \quad \text{and} \quad u^i x_i = 0. \tag{4}$$

We consider the non-static plane symmetric metric as

$$ds^2 = e^{2h}(dt^2 - dr^2 - r^2 d\theta^2 - S^2 dz^2) \tag{5}$$

where h and S are functions of t only. We now consider x^i to be along z-axis so that $x^i = (0, 0, 0, e^{-h}/S)$.

The field equations (1) and (2) with the help of (3) and (4) can be written as

$$\frac{\ddot{S}}{S} + 2\ddot{h} + \dot{h}^2 + \frac{2\dot{S}\dot{h}}{S} + \frac{1}{2}\dot{\phi}^2 = 0, \tag{6}$$

$$2\ddot{h} + \dot{h}^2 + \frac{1}{2}\dot{\phi}^2 = \lambda e^{2h}, \tag{7}$$

$$3\dot{h}^2 + \frac{2\dot{S}\dot{h}}{S} + \frac{1}{2}\dot{\phi}^2 = \rho e^{2h}, \tag{8}$$

$$\ddot{\phi} + \dot{\phi}\left(\frac{\dot{S}}{S} + 2\dot{h}\right) = 0, \tag{9}$$

where over head dot denotes partial derivative with respect to t.

From all the three energy conditions (weak, strong and dominant) for string model, one can find that $\rho > 0$ and $\rho_p \geq 0$ and the sign of λ is unrestricted [14].

3 Solution to the Field Equations

Equations (6)–(9) are four equations in five unknowns. Due to highly non-linear character of the field equations, we find the solutions of the field equations for the following three cases, viz., (i) $\rho = \lambda$ and (ii) $\rho = -\lambda$ and (iii) $\rho = (1+\omega)\lambda$ where $\omega > 0$.

Case 1. The case $\rho = \lambda$ refers to geometric strings. Now (6)–(9) reduce to

$$\ddot{h} + \frac{a_1^2}{2} e^{-h} = 0 \tag{10}$$

which on integration yields

$$e^h = \operatorname{Sinh} t \tag{11}$$

where we have assumed, without loss of generality, the constant $a_1 = 2$. Thus the general solution of the field equations (6)–(9) is given by

$$\begin{aligned} S &= \operatorname{Cosech} t, \\ e^h &= \operatorname{Sinh} t, \\ \phi &= \phi_0 \log(e^t - 1)/(e^t + 1). \end{aligned} \tag{12}$$

The non-static plane symmetric model for a cloud of geometric strings coupled with zero mass scalar fields can be expressed

$$ds^2 = \operatorname{Sinh}^2 t\,(dt^2 - dr^2 - r^2 d\theta^2 - \operatorname{Cosech}^2 t\, dz^2). \tag{13}$$

The energy density and tension density of strings are given by

$$\lambda = \rho = \operatorname{Cosech}^2 t \tag{14}$$

and physical and kinematical properties of the model are

The expansion scalar: $$\theta = u^i_{;i} = 2\operatorname{Cosech} t \operatorname{Coth} t \tag{15}$$

The spatial volume: $$V = \sqrt{-g} = r \operatorname{Sinh}^3 t \tag{16}$$

The shear scalar: $$\sigma^2 = \sigma^{ij}\sigma_{ij} = \frac{11}{9} \operatorname{Coth}^2 t \operatorname{Cosech}^2 t \tag{17}$$

The deceleration parameter: $$q = \frac{24\operatorname{Cosh}^4 t}{\operatorname{Sinh}^8 t}\left[\frac{\operatorname{Sinh} t\,(1 + \operatorname{Cosh}^2 t)}{\operatorname{Cosh}^2 t} - \frac{4}{6}\right]. \tag{18}$$

Equation (14) satisfies the energy conditions viz., $\rho \geq 0$, $\lambda \geq 0$ and $\rho^2 \geq \lambda^2$. The spatial volume of the model increases with the increase in time. It is also observed that $\frac{\sigma}{\theta} =$ constant, which indicate the model does not approach isotropy at any stage. The model given by (13) for a cloud of geometric strings possess a line singularity as ρ, λ, θ and σ tend to infinity and spatial volume tend to zero at initial epoch $t = 0$.

Case 2. When $\rho = -\lambda$, the field equations (6)–(9) admit the solution as

$$e^h = t^m,$$
$$S = t^{1-2m}, \quad \text{and} \tag{19}$$
$$e^\phi = kt^m$$

where the constants m and k are related by $(2+k^2)m^2 - 4m = 0$. From this relation we have either $m = 0$ or $m = \frac{4}{(k^2+2)}$. When $m = 0$, the field equations reduce to usual Einstein field equations without strings.

Hence in this case the plane symmetric non static metric take the form

$$dS^2 = t^{2m}(dt^2 - dr^2 - r^2 d\theta^2 - t^{2-4m} dz^2) \tag{20}$$

where $m = \frac{4}{(k^2+2)}$.

Now the string energy density ρ, tension density λ, the particle density ρ_p, the scalar expansion θ, the shear scalar σ, spatial volume V and the deceleration parameter q are given by

$$\rho = -\lambda = \frac{4m - 3m^2}{2t^{2m+2}}, \tag{21}$$

$$\rho_p = \frac{4m - 3m^2}{t^{2m+2}}, \tag{22}$$

$$\theta = \frac{(m+1)}{t^{m+1}}, \tag{23}$$

$$\sigma^2 = \frac{14m^2 - 8m + 5}{9t^{2m+2}}, \tag{24}$$

$$V = rt^{2m+1}, \tag{25}$$

$$q = \frac{(m+1)^4(3t^m - 1)}{t^{4m+4}}. \tag{26}$$

In this model, the spatial volume starts expanding while the expansion scalar and shear scalar decreases with the increase in cosmic time t. Since $q > 0$, there is no inflation at any stage. It is also observed that with the increase in cosmic time, ρ and λ vanish identically for large values of time t i.e., $\rho \to 0$ and $\lambda \to 0$ as $t \to \infty$ which shows that the model is a vacuum model. From (21) and (22) we note that $\frac{\rho_p}{|\lambda|} > 1$, thus we may conclude that the particles dominate over the strings in this model. For $0 < m < 4/3$, the energy conditions $\rho > 0$, $\lambda < 0$ and $\rho_p > 0$ are satisfied identically. If we set $m = 1$, we find that $\rho = -\lambda = \frac{1}{2t^4}$ and $q = \frac{16(3t-1)}{t^8}$ where as $\rho = \lambda = 0$ for $m = 4/3$ which again shows that the field equations reduce to vacuum case in the presence of zero-mass scalar fields.

Case 3. The P-strings are represented by $\rho = (1+\omega)\lambda$, where $\omega > 0$, which is also known as the Takabayasi strings. In this case (6)–(9) admit the following solution

$$S = (a_1 t + b_1)^{-1/(1+\omega)},$$
$$e^{2h} = (a_1 t + b_1)^{(1+\omega)/(2+\omega)}, \quad \text{and} \tag{27}$$

$$e^{\phi} = \phi_0(a_1 t + b_1)^{a_2/a_1} \tag{28}$$

where a_1, a_2, b_1 and ϕ_0 are arbitrary constants and are connected by

$$a_1^2(2+\omega)(2+3\omega) - 2a_2^2(1+\omega)^2 = 0. \tag{29}$$

By making use of (27) and (28) together with (29) in (7) and (8), the string energy density ρ and tension density λ vanish identically. This shows that the P-strings or Takabayasi strings do not co-exist with zero mass scalar fields in general relativity.

4 Conclusions

We have obtained the solutions of Einstein field equations in the presence of zero-mass scalar field coupled with a cloud of massive strings. It is observed that the non-static plane symmetric model had a line singularity when the string tension density equal to its energy density, where as the model reduces to a vacuum model for large values of t in case of the sum of string energy density and tension density equals to zero. However the solution exists in a special case i.e., for $0 < m < 4/3$. Also it is observed that the p-strings or Takabayasi strings vanish in the presence of zero-mass scalar fields.

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